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II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Squaring the equation as it stands, we get $2x+1+2\sqrt{x(x+1)}=\frac{9}{x+1}$.

Clearing of fractions and leaving the radical by itself in the first member, we get $2(x+1)\sqrt{x(x+1)}=8-3x-2x^2$. Squaring, arranging, and cancelling, we get the quadratic $35x^2+52x=64$, the two roots of which are $x=\frac{4}{5}$ and $-\frac{16}{7}$, the the former of which satisfies the equation $\sqrt{x+1}+\sqrt{x}=\frac{3}{\sqrt{x+1}}$, and the latter the equation $\sqrt{x+1}-\sqrt{x}=\frac{3}{\sqrt{x+1}}$.

Clearing the original equation of its denominator $\sqrt{x+1}$, we have $x+1+\sqrt{x(x+1)}=3$, or $\sqrt{x(x+1)}=2-x$. Squaring, we have $5x=4$. $\therefore x=\frac{4}{5}$.

III. Solution by F. M. McGAW, A. M., Professor of Mathematics in Bordentown Military Institute, Bordentown, N. J.; CHAS. C. CROSS, Laytonsville, Md.; G. B. M. ZERR, A. M., Ph. D., The Russell College, Lebanon, Va.; and J. P. BURDETTE, Class of '97, Dickinson College, Carlisle, Pa.

(1), $\sqrt{x+1}+\sqrt{x}=\frac{3}{\sqrt{1+x}}$. $2x+1+2\sqrt{x+x^2}=\frac{9}{1+x}$.

$\therefore 8-2x^2-3x=2(x+1)\sqrt{x+x^2}$. $\therefore 35x^2+52x=64$. $\therefore x=\frac{4}{5}$, or $-2\frac{2}{7}$.

(2). Regarding $\sqrt{x+1}$ as affected by the ± sign

$\sqrt{x}=\frac{2-x}{\sqrt{1+x}}$ or $\frac{4+x}{\sqrt{1+x}}$.

$\therefore x=(4-4x+x^2)/(1+x)$, or $(16+8x+x^2)/(1+x)$.

$\therefore x=\frac{4}{5}$, or $x=-2\frac{2}{7}$.

Also solved by *A. H. BELL.*

75. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

Mr. B's farm is in shape a quadrilateral, both inscriptible and circumscriptible, and contains an area of $k=10752$ square rods. The square described on the radius of its inscribed circle contains $r^2=2304$ square rods; while the square described on the radius of its circumscribed circle contains an area of $R^2=7345$ square rods. Required the lengths of the sides of his farm.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

Let a, b, c, d be the sides required. By the conditions of the problem, $a+c=b+d$; $abcd=k^2=115605504$ ....(1).

$\frac{1}{2}r(a+b+c+d)=k$, or $a+b+c+d=2k/r=448$.

$\therefore a+c=b+d=k/r=224$ ....(2).

$$R=\frac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(bc+ad)}{abcd}}.$$

$\therefore\ (ab+cd)(ac+bd)(bc+ad)=16R^2k^2=13585958830080\ldots\ldots\ldots\ldots(3).$

Substituting (2) in (1) and (3), we get

$$(224a-a^2)(224b-b^2)=115605504\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

$$\{ab+(224-a)(224-b)\}\{a(224-a)+b(224-b)\}\{b(224-a)+a(224-b)\}$$
$$=13585958830080\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5).$$

Eliminating b from (4) and (5), we get, after reducing and factoring,

$$(a-168)(a-128)(a-96)(a-56)=0.$$

$\therefore$ The sides are 168, 128, 96, 56 rods, respectively.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.; and COOPER D. SCHMITT, A. M., Professor of Mathematics in the University of Tennessee, Knoxville, Tenn.

Denoting the four consecutive sides of the quadrilateral by a, b, c, d, we have, from well-known geometrical formulæ and principles:

$abcd=k^2\ldots\ldots\ldots\ldots\ldots(1)$; $r^2=abcd/[a+c]^2=k^2/[a+c]^2\ldots\ldots\ldots\ldots\ldots(2)$;

$a+c=b+d=k/r\ldots\ldots(3)$; $R^2=\{[ac=bd][ad+bc][ab+cd]\}/16k^2\ldots\ldots(4).$

Putting $ac=x$, and $bd=y$, we have in (4),

$[x+y]\{ac[b^2+d^2]+y[a^2+c^2]\}=16k^2R^2$; or

$[x+y]\{x[(k^2/r^2)-2y]+y[(k^2/r^2)-2x]\}=16k^2R^2$; or

$[x+y]\{[x+y][k^2/r^2]-4xy\}=16k^2R^2$, and since $xy=abcd=k^2$,

$[x+y]\{[x+y]-4r^2\}=16R^2r^2$; or, reduced

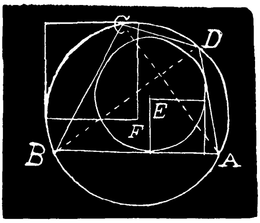
$[x+y]^2-4r^2[x+y]=16R^2r^2$; whence $x+y=2r^2+2r\sqrt{r^2+4R^2}$,

and combining this with $xy=k^2$, we find x and y. Thus, we find for the given numerical values $x+y=21696$, $xy=115605504$, whence $x=12848$, $y=9408$. Now we have $ac=12288$, $a+c=224$, and $bd=9408$, $b+d=224$.

Whence $a=128$, $c=96$, $b=168$, $d=56$.

III. Solution by the PROPOSER.

Let $ABCD$ represent the farm, and let $x=CD$, $y=DA$, $z=AB$, $w=BC$, in order. We have $x+z=y+w\ldots\ldots\ldots\ldots\ldots\ldots(1)$. Also the following, where $s=x+y+z+w$, the perimeter of the quadrilateral :



$k^2=\frac{1}{16}\{[s-2x][s-2y][s-2z][s-2w]\}\ldots\ldots\ldots\ldots(2)$;

$rs=2k\ldots\ldots\ldots\ldots\ldots(3)$; $k^2=xyzw\ldots\ldots\ldots\ldots\ldots(4)$;

$$R^2=\{[xy+zw][xz+yw][xw+yz]\}$$
$$\div\{[s-2x][s-2y][s-2z][s-2w]\}\ldots\ (5).$$

Substitute in (2), (4), and (5) :

$m=xy+xz+xw+yz+yw+zw$;

$n=xyz+xyw+xzw+yzw$; and $p=xyzw$.

Then we shall have by involving terms and re-factoring,

$$16k^2=16p-s^4+4s^2m-8sn\dots\dots(6)\ ;\quad k^2=p\dots\dots(7)\ ;$$

$$R^2=16p-s^4+4s^2m-8sn=n^2-4pm+ps^2\dots\dots(8).$$

From (3), (6), (7), and (8), we obtain by elimination and resolution,

$$s=2k/r=448\ ;\quad m=[k^3+r^3n]/r^2k=71872\ ;\quad n=2rk+2k\sqrt{[4R^2+r^2]}=4859904\ ;$$

$$p=k^2=115605504.$$

We now, by the "Theory of Equations," construct the biquadratic, the four roots of which will be the values of x, y, z, and w.

$$x^4-448x^3+71872x^2-4859904x=-115605504\dots\dots(9).$$

The four roots of equation (9), we find to be 56, 96, 128, and 168. Arranging these values in conformity with equation (1), we have, $CD=x=56$ rods, $DA=y=96$ rods, $AB=z=168$ rods, and $BC=w=128$ rods.

IV. Solution by A. H. BELL, Hillsboro, Illinois.

Since circumscriptible quadrilaterals have the sums of their opposite sides equal, take $x+y$, $x+z$, $x-y$, $x-z$, for the sides AB, BC, DC, and AD.

$$\therefore\ 2rx=k,\ x=k/2r\dots\dots(1).$$

$$\overline{BD}^2=[x+y]^2+[x-z]^2-2[x+y][x-z]\cos A\dots\dots(3).$$

$$\overline{BD}^2=[x+z]^2+[x-y]^2+2[x-y][x+z]\cos A,$$

$$\{\cos C=\cos[180-A]=-\cos A\}\dots\dots(4).$$

$$\therefore\ \cos A=\frac{x[y-z]}{x^2-y^2}.\quad \sin^2A=1-\cos^2A=\frac{[x^2-y^2][x^2-z^2]}{[x^2-y^2]^2}\dots\dots(5).$$

$$\overline{BD}^2=\frac{\{[x^2-y^2]+[x^2-z^2]\}[x^2+y^2]}{[x^2-y^2]}\ ;\ \text{also}\ R^2=\frac{\overline{BD}^2}{4\sin^2A}\dots\dots(6).$$

$$2k=[x+y][x-z]\sin A+[x-y][x+z]\sin A,\ \text{or}\ [x^2-y^2]\sin A=k\dots\dots(7).$$

Substituting the value of $\sin A$, (5) in (7), and

$$[x^2-y^2][x^2-z^2]=k^2\dots\dots(8),$$

Then (6) becomes, $[x^2-y^2+x^2-z^2][x^4-y^2z^2]=4Rk^2\dots\dots(9).$

Let the product of the opposite sides $v=x^2-y^2$. $\therefore\ y^2=x^2-v\dots(10)$;

and $w=x^2-z^2$. $\therefore\ z^2=x^2-w\dots\dots(11).$

Then (8), and (1), $vw=k^2=4r^2x^2\dots\dots(12)$;

and (9) reduces to $[v+w]^2-4r^2[v+w]=16R^2r^2\dots\dots(13).$

$\therefore\ v+w=2r^2\pm2r\sqrt{4R^2+r^2}$ ....(14).

$(14)^2-4(7)$, etc. $v-w=2r[4R^2+2r^2-4x^2\pm2r\sqrt{4R^2+r^2}]^{\frac{1}{2}}$ ....(15).

(14)±(15) after substituting the given values, $v=12288$, and $w=9408$.

(1), (5), and (6) $x=112$, $y=56$, and $z=16$, and the required sides AB, BC, CD, and AD are 168 rods, 128 rods, 56 rods, and 96 rods, respectively.

Also $BD=158.22$ rods.

Also solved by *CHARLES C. CROSS* and *H. C. WILKES*.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

61. Proposed by W. H. CARTER, Professor of Mathematics, Centenary College of Louisiana, Jackson, La.

If $r=a\sin n\theta$ is the polar equation of a curve, show (1) that the curve consists of n or $2n$ loops according as n is an odd or an even integer; (2) that its area is $\frac{1}{4}$ or $\frac{1}{2}$ of the circumscribing circle according as n is an odd or an even integer.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

$r=a\sin n\theta$. Let $r=0$, then $\sin n\theta=0$.

$\therefore\ \theta=0,\ 2\pi/n,\ 4\pi/n,\ 6\pi/n,\ 8\pi/n,\ \ldots\ldots$, are the angles at which the the curve cuts the polar axis at the pole.

$dr/d\theta=na\cos n\theta=0$. $\therefore\ \theta=\pi/2n,\ 3\pi/2n,\ 5\pi/2n,\ 7\pi/2n,\ \ldots\ldots$, gives the points where r has its greatest value, namely, $\pm a$.

When n is odd the values of $n\theta$ for the angles $0,\ 2\pi/n,\ 4\pi/n,\ 6\pi/n,\ 8\pi/n,\ \ldots\ldots$, are $0,\ 2\pi,\ 4\pi,\ 6\pi,\ 8\pi,\ \ldots\ldots$

When n is even the values of $n\theta$ for the angles $0,\ 2\pi/n,\ 4\pi/n,\ 6\pi/n,\ 8\pi/n,\ \ldots\ldots$, are $0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ \ldots\ldots$

$\therefore$ When n is even the polar axis is cut, at the pole, $2n$ times, but only n times when n is odd.

A = area of one loop.

$$A=\tfrac{1}{2}a^2\int_0^{\pi/n}\sin^2 n\theta d\theta,\ =\frac{\pi a^2}{4n}.$$

$$\therefore\ \frac{\pi a^2}{4n}\times n=\frac{\pi a^2}{4},\ \text{for } n \text{ odd};\ \frac{\pi a^2}{4n}\times 2n=\frac{\pi a^2}{2},\ \text{for } n \text{ even}.$$